“Condensed Matter Theory”

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Winter semester 2018/19 (30h) / IFPAN, room C, 8:30 / First Lecture: 2.10.2018

Plan of the course:

1. **Free particle** [1]
   1.0. Schroedinger equation
   1.1. Free particle in a box
   1.2. Free particle on a lattice: Bloch function
   1.3. Tight-binding approximation
2. **Many, non-interacting particles** [1]
   2.1. Interchanging identical particles
   2.2. Pauli exclusion principle
   2.3. Thermal distributions of fermions and bosons
   2.4. Wave function for free fermions: Slater determinant
3. **Second quantization**
   3.1. Occupation representation [1]
   3.2. Field operators [1]
   3.3. Single-particle operators in II quantization (particle number, spin) [1]
   3.4. Two-particle operators: interaction between electrons (Hubbard model and beyond) [1]
   3.5. Phonons [3]
      3.5.a Classical ion lattice vibrations (mono- and di- atomic chains)
      3.5.b ***Interlude:*** Harmonic oscillator in brief
      3.5.c Quantum ion lattice vibrations
   3.6. Electron-phonon interaction [3]
4. **Quasi-particles**
   4.1. Band structure effects (e.g. heavy fermions)
   4.2. ***Interlude:*** Canonical transformations
   4.3. Polaron (Froehlich model) [5]
      4.3.a Infinite mass impurity
      4.3.b Finite mass impurity
   4.4. Fermi liquid [4]
      4.4.a Observations for liquid ^{3}He
      4.4.b Phenomenological Landau’s theory
5. ***Interlude:*** **Single-particle Green’s function for fermions** [2, 4]
   5.1. Definition
   5.2. Green’s function for free electron
   5.3. Spectral representation
   5.4. Microscopic foundations of Fermi liquid
   5.5. Dyson equation and self-energy
   5.6. Low-energy sector of Hubbard model as Fermi liquid
6. **Mott’s metal to insulator transition**
   6.1. Mott-Wigner’s localization criterion [1]
   6.2. One-site Hubbard model [8]
   6.3. t-J model [1,6] (original derivation http://th-www.if.uj.edu.pl/ztms/download/jSpalek/tJmodel.pdf)
   6.4. ***Interlude:*** Wick’s theorem for Slater determinant in practice
   6.5. Gutzwiller approximation: Fermi-liquid and Brinkman-Rice transition [7]
   6.6. Gutzwiller approximation combined with variational Schrieffer-Wolff transformation [6]

7. **Magnetism**
   7.1. Concept of a spontaneously broken symmetry
   7.2. ***Interlude:*** mean-field approximation **[8]**
   7.3. Itinerant ferromagnetism in Hubbard model, Stoner criterion **[1,8]**
   7.4. ***Interlude:*** bipartite lattice and particle-hole transformation **[8]**
   7.5. Itinerant antiferromagnetism in Hubbard model
   7.6. Magnetism of localized spins
      7.6.a Anderson impurity model and Kondo interaction **[2]**
      7.6.b Dirac's exchange interaction **[1]**
8. **Conventional superconductivity**
   8.1. Effective electron-electron attraction mediated by phonons **[9]**
   8.2. Cooper problem **[10]**
   8.3. Bardeen-Cooper-Schrieffer (BCS) theory **[10]**
      8.3.a BCS variational wave function
      8.3.b Mean-field approach
   8.4. Quasiparticle and pair tunneling: Microscopic origin of the Josephson effect **[10]**
9. **Basis of density functional theory** **[Last lecture led by dr Carmine Autieri]**
   9.1. Hohenberg and Kohn theorem
   9.2. Kohn-Sham equations
   9.3. LDA and GGA: successes and failures

Suggested literature:
[1] J. Spałek "Wstęp do fizyki materii skondensowanej"
[2] P. Coleman "Introduction to Many-Body Physics"
[3] H. Bruus, K. Flensberg "Many-body quantum theory in condensed matter physics"
[4] A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinski "Methods of Quantum Field Theory in Statistical Physics"
[5] T. Devreese, arxiv:1611.06122
[6] M. Wysokiński, M. Fabrizio, Phys. Rev. B **95**, 161106 (2017)
[7] W. F. Brinkman and T. M. Rice, Phys. Rev. B **2**, 4302 (1970)
[8] R. Scalettar, Lecture notes of the Autumn School on Correlated Electrons 2016 "An Introduction to the Hubbard Hamiltonian", (Ch. 4)
(https://www.cond-mat.de/events/correl16/manuscripts/scalettar.pdf)
[9] R. Heid, Lecture notes of the Autumn School on Correlated Electrons 2017 "Electron-Phonon Coupling", (Ch. 4)
(https://www.cond-mat.de/events/correl17/manuscripts/heid.pdf)
[10] J. B. Ketterson, S. N. Song, "Superconductivity"

Exam:

Date of the oral exam is scheduled personally through e-mail (wysokinski@magtop.ifpan.edu.pl). Examinee will be asked to answer one of the problems listed below after 15-minute-long preparation. It is allowed to bring all possible notes or books that can help in preparing answer on separate sheet of paper that can be subsequently used while presenting answer.

Problems in the pool concerns (mostly derivation of):

1) electron-phonon interaction
2) momentum-independent energy (E_0) of polaron with finite mass impurity.
3) effective-mass renormalization in Landau theory of the Fermi liquid.
4) transformation to t-J model
5) Brinkman-Rice transition in Hubbard model
6) Stoner criterion for ferromagnetism in Hubbard model
7) mean-field Hamiltonian for antiferromagnetism in Hubbard model
8) Kondo interaction from Anderson impurity model
9) attractive electron-electron interaction in Froehlich model
10) energy of bound state in Cooper problems
11) isotropic gap function in BCS theory at zero temperature
12) critical temperature of superconductivity in BCS theory
13) microscopic origin of the Josephson effect